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Lebanese University
Faculty of information
Section 1

Final Exam: Discrete Mathematics **Time: 90 minutes**

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Problem 1.

1. For each of the following circle an answer indicating if the proposition is either a tautology, a contradiction, or a contingency.
   a) $(a \wedge b) \rightarrow a$
   b) $(a \vee b) \rightarrow (a \wedge b)$
   c) $(a \oplus b) \rightarrow (a \vee b)$

2. Determine whether each of these compound propositions is satisfiable.
   a) $\neg(p \vee q) \wedge (p)$
   b) $\neg p \wedge \neg q \wedge p$

3. Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

Problem 2.

1. Let A, B and C are sets. Show that:

$$(B - C) \cap (A - C) = (A \cap B) - C$$

2. Let f be a function from the set A to the set B. Let S and T be subsets of A. Show that

$$f(S \cup T) \subseteq f(S) \cup f(T)$$

3. Find $f \circ g$ and $g \circ f$ where $f(x) = x^2 + 1$ and $g(x) = \sqrt{x+3}$
4. Let f be the function from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = x^3 - 1$. Find
   a) $f^{-1}(\{0\})$
   b) $f^{-1}(\{x/x > 7\})$
   c) $f^{-1}(\{x/0 < x < 26\})$

Problem 3.

1. Show that if a, b, and m are integers such that $m \geq 2$ and $a \equiv b \pmod{m}$ then $\gcd(a, m) = \gcd(b, m)$
2.
   a) Convert the binary expansion of the integer to an octal expansion.

$$(1000011011)_2$$

   b) Convert the decimal expansion of each of the integer to a binary expansion.

$$4532$$

3.

a) Find an inverse of a modulo m for each of these pairs of relatively prime integers.

$$a = 17, m = 43.$$

b) Solve each of these congruences using the modular inverses found in part (a)

$$17x \equiv 9 \pmod{43}$$

Problem 4.

1. Consider the "equals" relation on the set of all positive integers $R = \{(a, b) / \ a = b\}$ be the relation on the set of real numbers.
   Show that the relation R is an equivalence relation on the set of integers.

2. Let R_1 and R_2 be the "congruent modulo 4" and "the congruent modulo 9" relations, respectively, on the set of integers. That is,

$$R_1 = \{(a, b) / a \equiv b \pmod 4\}$$
$$R_2 = \{(a, b) / a \equiv b \pmod 9\}$$

   Find
   a) $R_1 \cup R_2$
   b) $R_1 \cap R_2$
   c) $R_2 - R_1$
   d) $R_1 \oplus R_2$

3. Let R_1 and R_2 be relations on a set A represented by the matrices

$$M_{R_1} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \qquad M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

   Find the matrix representing
   a) $R_1 \cup R_2$
   b) $R_1 \cap R_2$
   c) $R_1 \circ R_2$
   d) $\overline{R_1}$ and R_2^{-1}

Good Luck